GCE Examinations

Advanced Subsidiary / Advanced Level

Decision Mathematics
Module D2

Paper A

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

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D2 Paper A – Marking Guide

1.

		B I	II	III	row minimum
A	I	$^-3$	4	0	$^-3$
	II	2	2	1	1
	III	3	$^-2$	$^-1$	$^-2$
column maximum		3	4	1	

M1 A1

max (row min) = min (col max) = 1 $\therefore$ saddle point — M1

$\therefore$ *A* should play II all the time, *B* should play III all the time — M1 A1 **(5)**

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2. *(a)*
x_{11} – number of crates from *A* to *D*
x_{12} – number of crates from *A* to *E*
x_{13} – number of crates from *A* to *F*
x_{21} – number of crates from *B* to *D*
x_{22} – number of crates from *B* to *E*
x_{23} – number of crates from *B* to *F*
x_{31} – number of crates from *C* to *D* — B1
x_{32} – number of crates from *C* to *E*
x_{33} – number of crates from *C* to *F*

(b) minimise
$z = 19x_{11} + 22x_{12} + 13x_{13} + 18x_{21} + 14x_{22} + 26x_{23} + 27x_{31} + 16x_{32} + 19x_{33}$ — B2

(c)
$x_{11} + x_{12} + x_{13} = 42$ number of crates at *A*
$x_{21} + x_{22} + x_{23} = 26$ number of crates at *B*
$x_{31} + x_{32} + x_{33} = 32$ number of crates at *C*
$x_{11} + x_{21} + x_{31} = 29$ number of crates required by *D*
$x_{12} + x_{22} + x_{32} = 47$ number of crates required by *E* — M1 A1
$x_{13} + x_{23} + x_{33} = 24$ number of crates required by *F*
$x_{ij} \geq 0$ for all i, j
reference to balance — B1 **(6)**

3.

Stage	State	Destination	Cost	Total cost	
1	Marquee	Deluxe	20	20*	
		Cuisine	24	24	
	Castle	Deluxe	21	21	
		Castle	15	15*	
		Cuisine	22	22	
	Hotel	Deluxe	18	18*	
		Cuisine	23	23	M1 A1
		Hotel	19	19	
2	Church	Marquee	2	2 + 20 = 22	
		Castle	5.5	5.5 + 15 = 20.5*	
		Hotel	3	3 + 18 = 21	
	Castle	Marquee	3	3 + 20 = 23	
		Castle	5	5 + 15 = 20*	
	Registry Office	Marquee	3.5	3.5 + 20 = 23.5	
		Castle	6	6 + 15 = 21	M1 A2
		Hotel	2	2 + 18 = 20*	
3	Home	Castle	3	3 + 20.5 = 23.5	
		Church	5	5 + 20 = 25	
		Registry	1	1 + 20 = 21*	A1

minimum cost with
ceremony – Registry Office
reception – Hotel
catering – Deluxe — M1 A1

cost = £2100 — A1 **(9)**

4. (i)

order:	1	4	8	2	3	6	5	7
	A	*B*	*C*	*D*	*E*	*F*	*G*	*H*
A	–	85	$\boxed{59}$	31	47	52	74	41
B	85	–	104	73	$\boxed{51}$	68	43	55
C	59	104	–	54	62	88	61	$\boxed{45}$
D	$\boxed{31}$	73	54	–	40	59	65	78
E	47	51	62	$\boxed{40}$	–	56	71	68
F	52	68	88	59	56	–	$\boxed{53}$	49
G	74	$\boxed{43}$	61	65	71	53	–	63
H	41	55	45	78	68	$\boxed{49}$	63	–

M1 A2

tour: *ADEBGFHCA*

upper bound = 31 + 40 + 51 + 43 + 53 + 49 + 45 + 59 = 371 km A1

(ii) e.g. beginning at *A*

order:	1	4	7	2	3	6	5	
	A	*B*	*C*	*D*	*E*	*F*	*G*	*H*
A	–	85	59	31	47	52	74	41
B	85	–	104	73	$\boxed{51}$	68	43	55
C	59	104	–	$\boxed{54}$	62	88	61	45
D	$\boxed{31}$	73	54	–	40	59	65	78
E	47	51	62	$\boxed{40}$	–	56	71	68
F	$\boxed{52}$	68	88	59	56	–	53	49
G	74	$\boxed{43}$	61	65	71	53	–	63
H	41	55	45	78	68	49	63	–

M1 A2

weight of MST = 31 + 40 + 51 + 43 + 52 + 54 = 271 A1

lower bound = weight of MST + two edges of least weight from *H*

= 271 + 41 + 45 = 357 km M1 A1

$\therefore$ 357 $\leq d \leq$ 371 A1 **(11)**

5. *(a)* let X play strategies X_1 and X_2 with proportions p and $(1 - p)$

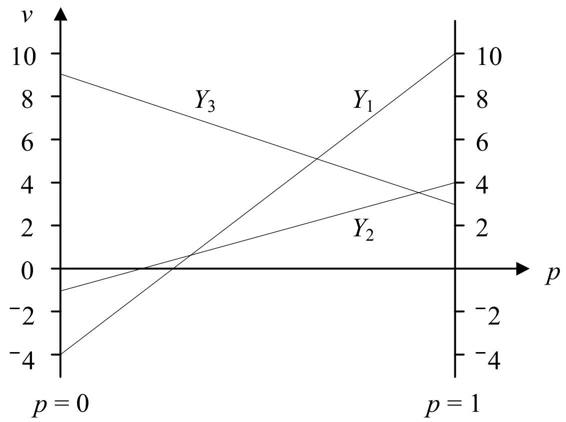
expected payoff to X against each of Y's strategies:

Y_1 $\quad 10p - 4(1 - p) = 14p - 4$

Y_2 $\quad 4p - (1 - p) = 5p - 1$ M1 A1

Y_3 $\quad 3p + 9(1 - p) = 9 - 6p$

giving



B2

it is not worth player Y considering strategy Y_1 B1

for optimal strategy $5p - 1 = 9 - 6p$

$\therefore 11p = 10, \ p = \frac{10}{11}$

$\therefore$ X should play X_1 $\frac{10}{11}$ of time and X_2 $\frac{1}{11}$ of time M1 A1

(b) let Y play strategies Y_2 and Y_3 with proportions q and $(1 - q)$

expected loss to Y against each of X's strategies:

X_1 $\quad 4q + 3(1 - q) = q + 3$

X_2 $\quad {}^{-}q + 9(1 - q) = 9 - 10q$ M1 A1

for optimal strategy $q + 3 = 9 - 10q$

$\therefore 11q = 6, \ q = \frac{6}{11}$

$\therefore$ Y should not play Y_1, should play Y_2 $\frac{6}{11}$ of time and Y_3 $\frac{5}{11}$ of time M1 A1

(c) value $= (5 \times \frac{10}{11}) - 1 = 3\frac{6}{11}$ M1 A1 **(13)**

6. need to maximise so subtract all values from 55 giving M1

$$\begin{array}{cccc|c} & & & & \text{row min.} \\ 18 & 26 & 11 & 4 & 4 \\ 10 & 25 & 12 & 14 & 10 \\ 23 & 28 & 16 & 5 & 5 \\ 12 & 30 & 4 & 0 & 0 \end{array}$$

reducing rows gives:

$$\begin{array}{lcccc} & 14 & 22 & 7 & 0 \\ & 0 & 15 & 2 & 4 \\ & 18 & 23 & 11 & 0 \\ & 12 & 30 & 4 & 0 \\ \hline \text{col min.} & 0 & 15 & 2 & 0 \end{array}$$

M1 A1

reducing columns gives:

$$\begin{array}{cccc} 14 & 7 & 5 & 0 \\ 0 & 0 & 0 & 4 \\ 18 & 8 & 9 & 0 \\ 12 & 15 & 2 & 0 \end{array}$$

M1 A1

2 lines required to cover all zeros, apply algorithm B1

$$\begin{array}{cccc} 12 & 5 & 3 & 0 \\ 0 & 0 & 0 & 6 \\ 16 & 6 & 7 & 0 \\ 10 & 13 & 0 & 0 \end{array}$$

(N.B. a different choice of lines will lead to the same final assignment) M1 A1

3 lines required to cover all zeros, apply algorithm

$$\begin{array}{cccc} 7 & 0^* & 3 & 0 \\ 0^* & 0 & 5 & 11 \\ 11 & 1 & 7 & 0^* \\ 5 & 8 & 0^* & 0 \end{array}$$

M1 A1

4 lines required to cover all zeros so allocation is possible B1

R_1 goes to A_2
R_2 goes to A_1
R_3 goes to A_4
R_4 goes to A_3 M1 A1 **(13)**

7. *(a)*

	W_A	W_B	W_C	Available
W_1	5	5		10
W_2		7	1	8
W_3			7	7
Required	5	12	8	

M1 A1

(b) taking $R_1 = 0$, $R_1 + K_1 = 7 \therefore K_1 = 7$ $R_1 + K_2 = 8 \therefore K_2 = 8$
$R_2 + K_2 = 6 \therefore R_2 = {}^-2$ $R_2 + K_3 = 5 \therefore K_3 = 7$ M1 A2
$R_3 + K_3 = 7 \therefore R_3 = 0$

	$K_1 = 7$	$K_2 = 8$	$K_3 = 7$
$R_1 = 0$	0	0	10
$R_2 = {}^-2$	9	0	0
$R_3 = 0$	11	5	0

improvement indices, $I_{ij} = C_{ij} - R_i - K_j$

$\therefore$ $I_{13} = 10 - 0 - 7 = 3$
$I_{21} = 9 - ({}^-2) - 7 = 4$
$I_{31} = 11 - 0 - 7 = 4$ M1 A1
$I_{32} = 5 - 0 - 8 = {}^-3$

(c) applying algorithm

	W_A	W_B	W_C
W_1	5	5	
W_2		$7 - \theta$	$1 + \theta$
W_3		θ	$7 - \theta$

let $\theta = 7$, giving

	W_A	W_B	W_C
W_1	5	5	
W_2			8
W_3		7	

M1 A1

no. of rows + no. of cols – 1 = 3 + 3 – 1 = 5
in this solution only 4 cells are occupied, less than 5 $\therefore$ degenerate B1

(d) placing 0 in (2, 2) so it is occupied
taking $R_1 = 0$, $R_1 + K_1 = 7 \therefore K_1 = 7$ $R_1 + K_2 = 8 \therefore K_2 = 8$
$R_2 + K_2 = 6 \therefore R_2 = {}^-2$ $R_2 + K_3 = 5 \therefore K_3 = 7$ M1 A1
$R_3 + K_2 = 5 \therefore R_3 = {}^-3$

	$K_1 = 7$	$K_2 = 8$	$K_3 = 7$
$R_1 = 0$	0	0	10
$R_2 = {}^-2$	9	0	0
$R_3 = {}^-3$	11	0	7

$\therefore$ $I_{13} = 10 - 0 - 7 = 3$
$I_{21} = 9 - ({}^-2) - 7 = 4$
$I_{31} = 11 - ({}^-3) - 7 = 7$ M1 A1
$I_{33} = 7 - ({}^-3) - 7 = 3$

all improvement indices are non-negative $\therefore$ pattern is optimal B1

5 lorries from W_1 to W_A, 5 lorries from W_1 to W_B,
8 lorries from W_2 to W_C, 7 lorries from W_3 to W_B A1

(e) total cost = $10 \times [(5 \times 7) + (5 \times 8) + (8 \times 5) + (7 \times 5)]$ = £1500 M1 A1 **(18)**

Total **(75)**

Performance Record – D2 Paper A

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	Total
Topic(s)	game, stable soln.	transport., formulate lin. prog.	dynamic prog., min.	TSP, nearest neighbour	game, graphical method	allocation, max.	transport., n-w corner, stepping-stone, degeneracy	
Marks	5	6	9	11	13	13	18	75
Student								